Enrollment No: ____________________ Exam Seat No: ____________________

C. U. SHAH UNIVERSITY

Summer Examination-2020

Subject Name : Engineering Mathematics - I

Subject Code : 4TE01EMT3 **Branch: B. Tech (All)**

Semester : 1 **Date : 26/02/2020** **Time : 02:30 To 05:30** **Marks : 70**

Instructions:

(1) Use of Programmable calculator & any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

Q-1 **Attempt the following questions:** **(14)**

a) If $y=\sin 2x\cos 2x$ then y_n equal to

(A) $\frac{1}{2}(4)^n\cos\left(\frac{n\pi}{2}+4x\right)$ (B) $\frac{1}{2}(4)^n\sin\left(\frac{n\pi}{2}+4x\right)$

(C) $\frac{1}{2}(4)^n\sin\left(\frac{n\pi}{2}+2x\right)$ (D) none of these

b) If $y=\frac{3x+2}{2x+3}$, then y_n equal to

(A) $\frac{5(-1)^{n+1}n!2^{n-1}}{(2x+3)^{n+1}}$ (B) $\frac{5(-1)^{n}n!2^{n}}{(2x+3)^{n+1}}$ (C) $\frac{5(-1)^{n}n!2^{n}}{(2x+3)^{n-1}}$

(D) none of these

c) If $y=\log(1+x)$, then x equal to

(A) $1+y+\frac{y^2}{2!}+\frac{y^3}{3!}$ (B) $1-\frac{y^2}{2!}+\frac{y^4}{4!}-.....$

(C) $y+\frac{y^2}{2!}+\frac{y^3}{3!}+\frac{y^4}{4!}+.....$ (D) none of these

d) $1+\frac{x^2}{2!}+\frac{x^4}{4!}+\frac{x^6}{6!}+.....\infty$ represent expansion of

(A) $\sinh x$ (B) $\cosh x$ (C) $\cos x$ (D) e^x

e) $\lim_{x\to 0}\frac{a^x-b^x}{x}=$ _____

(A) $a-b$ (B) $\log(a-b)$ (C) $\log ab$ (D) $\log\left(\frac{a}{b}\right)$

f) $\lim_{x\to\infty}\frac{x^n}{e^{ax}}$ (n being a positive integer and $a>0$) $=$ _____

(A) – 1 (B) 0 (C) 1 (D) None of these

g) If $P = r\tan\theta$, then $\dfrac{\partial P}{\partial r}$ is equal to

(A) $\sec^2\theta$ (B) $\tan\theta$ (C) $\tan\theta + r\sec^2\theta$ (D) $\dfrac{1}{2}\tan\theta$

h) If $f(x, y) = 0$, then $\dfrac{dy}{dx}$ is equal to

(A) $\dfrac{\partial f/\partial x}{\partial f/\partial y}$ (B) $\dfrac{\partial f/\partial y}{\partial f/\partial x}$ (C) $-\dfrac{\partial f/\partial y}{\partial f/\partial x}$ (D) $-\dfrac{\partial f/\partial x}{\partial f/\partial y}$

i) If $u(x, y, z) = 0$ then the value of $\dfrac{\partial x}{\partial y}\cdot\dfrac{\partial y}{\partial z}\cdot\dfrac{\partial z}{\partial x}$ is equal to

(A) 1 (B) – 1 (C) 0 (D) none of these

j) If $f_1 = \dfrac{vw}{u},\ f_2 = \dfrac{wu}{v},\ f_3 = \dfrac{uv}{w}$; then $\dfrac{\partial(f_1, f_2, f_3)}{\partial(u, v, w)}$ is equal to

(A) 0 (B) 1 (C) 3 (D) none of these

k) The value of $\sum_{k=1}^{10}\left(\sin\dfrac{2\pi k}{11} - i\cos\dfrac{2\pi k}{11}\right)$ is

(A) – 1 (B) 0 (C) – i (D) i

l) If $\left(\dfrac{1-i}{1+i}\right)^{100} = a + ib$, then

(A) $a = 2,\ b = -1$ (B) $a = 1,\ b = 0$ (C) $a = 0,\ b = 1$ (D) $a = -1,\ b = 2$

m) If A is a non-zero column vector $(n \times 1)$, then the rank of matrix AA^T is

(A) 0 (B) 1 (C) $n-1$ (D) n

n) An eigenvalue of a square matrix A is $\lambda = 0$. Then

(A) $|A| \neq 0$ (B) A is symmetric (C) A is singular

(D) A is skew-symmetric

Attempt any four questions from Q-2 to Q-8

Q-2 **Attempt all questions** **(14)**

a) If $y = \tan^{-1} x$ then find the value of $y_n(0)$. **(5)**

b) Expand $f(x) = \dfrac{e^x}{e^x + 1}$ in powers of x up to x^3 by Maclaurin's series. **(5)**

c) If $V = \dfrac{1}{r}$ where $r^2 = x^2 + y^2 + z^2$ then show that $V(x, y, z)$ satisfies Laplace's equation $\dfrac{\partial^2 V}{\partial x^2} + \dfrac{\partial^2 V}{\partial y^2} + \dfrac{\partial^2 V}{\partial z^2} = 0$. **(4)**

Q-3 **Attempt all questions** **(14)**

a) If $y = a\cos(\log x) + b\sin(\log x)$ then prove that **(5)**

$$x^2 y_{n+2} + (2n+1)xy_{n+1} + (n^2+1)y_n = 0 \ .$$

b) Prove that $(1+x)^x = 1 + x^2 - \dfrac{1}{2}x^3 + \dfrac{5}{6}x^4 - .....$ **(5)**

	c)	Evaluate: $\lim_{x\to 1}\left(\frac{x}{x-1}-\frac{1}{\log x}\right)$.	**(4)**
Q-4		**Attempt all questions**	**(14)**
	a)	Evaluate: $\lim_{x\to 0}\left(\frac{a^x+b^x+c^x+d^x}{4}\right)^{\frac{1}{x}}$	**(5)**
	b)	If $x=r\sin\theta\cos\phi,\ y=r\sin\theta\sin\phi,\ z=r\cos\theta$ then find $\frac{\partial(x,\ y,\ z)}{\partial(r,\ \theta,\ \phi)}$.	**(5)**
	c)	Expand $f(x)=x^4-11x^3+43x^2-60x+14$ in powers of $(x-3)$.	**(4)**
Q-5		**Attempt all questions**	**(14)**
	a)	If $u=\tan^{-1}\left(\frac{x^2+y^2}{x-y}\right)$ then show that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=\frac{1}{2}\sin 2u$.	**(5)**
	b)	Evaluate: $\lim_{x\to 0}\frac{a}{x^2}\left[\frac{\sin kx}{\sin lx}-\frac{k}{l}\right]$	**(5)**
	c)	If $y=\frac{x^4}{(x-1)(x-2)}$ then find y_n .	**(4)**
Q-6		**Attempt all questions**	**(14)**
	a)	Using the formula $R=\frac{E}{I}$, find the maximum error and percentage of error in R if $I=20$ with a possible error of 0.1 and $E=120$ with a possible error of 0.05 and $R=6$.	**(5)**
	b)	Expand $\sin^5\theta\cos^2\theta$ in a series of sines of multiples of θ .	**(5)**
	c)	Find the inverse of $A=\begin{bmatrix}1&2&3\\2&4&5\\3&5&6\end{bmatrix}$ by Gauss-Jordan reduction method.	**(4)**
Q-7		**Attempt all questions**	**(14)**
	a)	Find the eigenvalues and eigenvectors of the matrix $A=\begin{bmatrix}1&-2\\-5&4\end{bmatrix}$.	**(5)**
	b)	Find the continued product of all the values of $\left(\frac{1}{2}+i\frac{\sqrt{3}}{2}\right)^{\frac{3}{4}}$.	**(5)**
	c)	If $\tan(\alpha+i\beta)=x+iy$ then prove that $x^2+y^2+2x\cot 2\alpha=1$.	**(4)**
Q-8		**Attempt all questions**	**(14)**
	a)	Investigate for what values of λ and μ the equations $x+y+z=6,\ x+2y+3z=10,\ x+2y+\lambda z=\mu$, have (i) no solution (ii) a unique solution (iii) an infinite number of solutions.	**(5)**
	b)	Find the fourth roots of unity and sketch them on the unit circle.	**(5)**
	c)	Verify Caley-Hamilton theorem for the matrix $A=\begin{bmatrix}2&1&1\\0&1&0\\1&1&2\end{bmatrix}$.	**(4)**